Precalc Warm Up #14-2

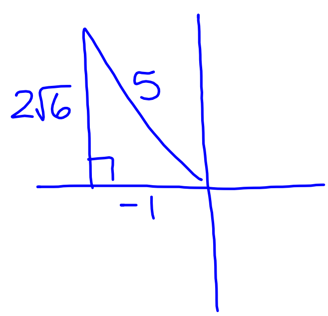
1. If $\cos\theta = -1/5$ and $\tan\theta < 0$, find $\csc\theta$

2. Solve $5^{2x^2-9x-5} = 1$

If $\cos\theta = -1/5$ and $\tan\theta < 0$, find $\csc\theta$

cosine negative in QII & III

tangent negative in QII & IV



$\csc \rightarrow \frac{h}{o}$

$\csc\theta = \frac{5}{2\sqrt{6}}$

$\boxed{\frac{5\sqrt{6}}{12}}$

2. Solve

a. $\log(3x-5) + \log(x+1) = 2$

b. $\log_x(1/25) = 2$

c. $3^{4x-9} = 19$

2. Solve

a. $\log(3x-5) + \log(x+1) = 2$

$\log(3x^2-2x-5) = 2$

$10^2 = 3x^2-2x-5$

$0 = 3x^2-2x-105$

b. $\log_x(1/25) = 2$

$x^2 = \frac{1}{25}$

$x = \pm\frac{1}{5}$

Base > 0 and $\neq 1$

$\boxed{\text{So } x = \frac{1}{5}}$

c. $3^{4x-9} = 19$

$\ln 3^{4x-9} = \ln 19$

$\frac{(4x-9)\ln 3}{\ln 3} = \frac{\ln 19}{\ln 3}$

$4x-9 = \frac{\ln 19}{\ln 3}$

* Don't calculate until after you isolate x!

Partial Fraction Decomposition Review

Type 1: Distinct linear factors
Write the partial fraction decomposition for $\frac{x+7}{x^2-x-6} = \frac{A}{(x-3)} + \frac{B}{(x+2)}$

$(x-3)(x+2)$

1) Factor denominator & write a separate fraction for each denominator.
2) Multiply by giant ones to make the denominators the same

$$\frac{x+7}{(x-3)(x+2)} = \frac{A(x+2)}{(x-3)(x+2)} + \frac{B(x-3)}{(x+2)(x-3)}$$

3) Write the "BASIC EQUATION": $x+7 = A(x+2) + B(x-3)$
4) Choose convenient values of x to find A & B

let $x=-2$: $-2+7 = A(-2+2) + B(-2-3)$

$5 = -5B \rightarrow B=-1$

let $x=3$: $3+7 = A(3+2) + B(0)$

$10 = 5A$

$A=2$

5) Write the decomposition

$$\boxed{\frac{x+7}{x^2-x-6} = \frac{2}{x-3} - \frac{1}{x+2}}$$

Type 2: Repeated linear factors
Write the partial fraction decomposition for $\frac{5x^2+20x+6}{x^3+2x^2+x} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$x(x^2+2x+1)$

$x(x+1)^2$

Each power of a repeat factor needs to be a denominator.

Basic Equation:

$5x^2+20x+6 = A(x+1)^2 + Bx(x+1) + Cx$

Let $x=0 \rightarrow 6=A$

Let $x=-1 \rightarrow 5-20+6 = C(-1)$

$-9 = -C \rightarrow C=9$

Let $x=1 \rightarrow 5+20+6 = 6(1+1)^2 + B(1)(1+1) + 9(1)$

$31 = 24 + 2B + 9$

$-2 = 2B \rightarrow B=-1$

$$\frac{5x^2+20x+6}{x^3+2x^2+x} = \frac{6}{x} - \frac{1}{x+1} + \frac{9}{(x+1)^2}$$

Type 3: Distinct linear and Quadratic Factors
Write the partial fraction decomposition for

$$\frac{3x^2+4x+4}{x^3+4x} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$x(x^2+4)$

A distinct quadratic factor is a quadratic that can't be factored. It gets a linear numerator: $Bx+C$

Basic Equation:

$3x^2+4x+4 = A(x^2+4) + x(Bx+C)$

let $x=0 \rightarrow$ $4 = A(4) \rightarrow A=1$

let $x=1$ w/ $A=1$ $\rightarrow 3+4+4 = (1)(1+4) + 1(B+C)$

$11 = 5 + B + C$

$6 = B + C$

Now choose another easy value for x, get another equation with B & C, then solve the system.

Type 4: Repeated Quadratic Factors
Write the partial fraction decomposition for

$$\frac{8x^3+13x}{(x^2+2)^2} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{(x^2+2)^2}$$

Each power of a repeat factor needs to be a denominator.

A distinct quadratic factor gets a linear numerator.

Basic Equation:

$8x^3+13x = (x^2+2)(Ax+B) + Cx + D$

* here you can see that $A=8$ since it will be the coefficient of the x^3

Keep going with your other tricks!

Type 5: Improper rational expressions
Write the partial fraction decomposition for

$$\frac{2x^3+x^2-7x+7}{x^2+2x-3} = 2x-3 + \frac{5x-2}{x^2+2x-3}$$

When the power of the numerator is higher than the denominator, divide first!

Now decompose the fraction part. :)

$$\begin{array}{r} 2x-3 \\ x^2+2x-3\,\overline{)\,2x^3+x^2-7x+7} \\ -(2x^3+4x^2-6x) \\ \hline -3x^2-x+7 \\ -(-3x^2-6x+9) \\ \hline 5x-2 \end{array}$$

$$\frac{4-x}{x^2+6x+8}$$

$$\frac{x^2+2x}{x^3-x^2+x-1}$$

In Exercises 65–72, write the partial fraction decomposition for the rational expression.

65. $\dfrac{4-x}{x^2+6x+8}$

$(x+4)(x+2)$

$$\frac{4-x}{(x+4)(x+2)} = \frac{A}{x+4} + \frac{B}{x+2}$$

$4-x = A(x+2) + B(x+4)$

let $x=-2 \rightarrow$ $4+2 = B(-2+4)$

$6 = 2B$

$B = 3$

$\left.\begin{array}{l}\text{Let } x=0 \\ \text{with } B=3\end{array}\right\}$ $4 = 2A + 3(4)$

$-8 = 2A$

$A = -4$

$$\boxed{\frac{4-x}{x^2+6x+8} = -\frac{4}{x+4} + \frac{3}{x+2}}$$

69. $\dfrac{x^2+2x}{x^3-x^2+x-1}$

$x^2(x-1)+1(x-1)$

$(x-1)\underbrace{(x^2+1)}_{}$

Non reducible quadratic needs a linear numerator

$$\frac{x^2+2x}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$x^2+2x = A(x^2+1) + (x-1)(Bx+C)$

☆ choose $x=1$ or $x=0$ to start find A, B & C.

$A = \frac{3}{2}$ $B = -\frac{1}{2}$ $C = \frac{3}{2}$

$$\frac{x^2+2x}{x^3-x^2+x-1} = \frac{3}{2x-2} + \frac{-x+3}{2x^2+2}$$

$$\underline{\qquad} - \frac{x-3}{2x^2+2}$$

Solve:

$\sec^2 x - 2 = 0$ on $[0, 2\pi)$

$\sec x = \pm\sqrt{2}$

$\cos x = \pm\frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Final Exam:

You may use my grapher and trig sheet and one page of notes with other formulas written in your own handwriting. No examples. You will turn them in after the final.

Bring: Pencils, eraser and something to do when you finish.

Final Exam covers:

PC 3.7, 3.8
PC 4, 5, 6
SL 7, 9, 12

Topics
- Partial Fraction Decomposition
- Rational Functions
- Exponents and Logarithms
- Trigonometry
- Vectors

Review:

Asymptotes for rational functions

Vertical asymptotes at the zeros of the denominator as long as they are not holes instead!

Horizontal Asymptotes: Compare the degree of the top to the degree of the bottom.

Slant Asymptotes: When the degree of the top is exactly one more than the degree of the bottom.

Asymptotes for rational functions

Vertical asymptotes at the zeros of the denominator as long as they are not holes instead!

$f(x) = \frac{x+1}{(x+1)(x-3)}$

hole @ $x=-1$

Vert @ $x=3$

Horizontal Asymptotes: Compare the degree of the top to the degree of the bottom.

$\frac{(\ )^m}{(\ )^n}$

$m = n \longrightarrow y = \frac{LC}{LC}$

$m < n \longrightarrow y = 0$

$m > n \longrightarrow$ No horiz.

$f(x) = \frac{3x-2}{x+5}$

horiz @ $y = 3$

vert @ $x = -5$

Slant Asymptotes: When the degree of the top is exactly one more than the degree of the bottom. $\longrightarrow$ Do $\div$

$f(x) = \frac{2x^2 - 5x + 4}{x-3}$

$\searrow (x-k)$

$k \nearrow$

$$\begin{array}{r|rrr} 3 & 2 & -5 & 4 \\ & & 6 & 3 \\ \hline & 2 & 1 & 7 \end{array}$$

ignore remainder

$y = 2x + 1$

slant asymptote

Answers to Final Review

Ch. 3 + 4

0. $-\frac{3}{x+4} + \frac{2}{x-3}$

1. $x = -1$, $y = x + 2$

2. d
3. a
4. b
5. b
6. c
8. d
9. a
10. a
11. b
12. d
13. a
14. b

Ch. 5

1. b
3. d

Ch. 5 cont.

4. d
5. d
6. c
7. c
8. a
9. d
10. b
11. b
12. a
13. c
14. d
15. a

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17. d
18. a
19. d
20. c

Chapter 6

1. d
2. b